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Plane waves do not polarize the vacuum†

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Abstract. Gravitational plane waves, like their electromagnetic and Yang–Mills counterparts, are undistorted by vacuum polarization effects to all loop orders, if no cosmological term is induced.

Virtual quantum effects generally alter the characteristics of the vacuum in which a classical external field propagates. In electrodynamics these polarization effects due to virtual charged pairs are expressible in terms of field-dependent dielectric properties of the vacuum (Heisenberg and Euler 1936, Weisskopf 1936). For the special case of a plane wave, however, Schwinger (1951) showed that its propagation is unaffected (to all orders) by virtual electron pairs; only an amplitude renormalization occurs. We shall give a simple argument for the general electromagnetic plane wave case, and show that for similar reasons a classical gravitational plane wave remains undistorted to all orders by arbitrary virtual pair effects (including gravitons). This rather unique property will be traced to the vanishing of all relevant invariants for these idealized solutions.‡

Our basic observation is that the effective Lagrangian describing the vacuum polarization must be a gauge invariant scalar function of the field strengths and their derivatives. Thus in the Maxwell case, it would be a power series of the form

$$\mathscr{L}^{M}_{\text{eff}} \sim \sum F \ldots \partial F \ldots \partial^{*} F \ldots \Box F \ldots \tag{1}$$

where $^{*}F$ is the dual tensor and derivative indices contract either with F's or with each other to form D'Alembertians as indicated. The overall coefficients will be appropriate powers of α and of the Compton wavelengths of the virtual particles times numerical coefficients. But the characteristic of a plane wave is that its field strength is of the form

$$F_{\mu\nu} = f_{\mu\nu} F(n_{\mu} x^{\mu})$$

where n_{μ} is a null vector and the constant amplitudes $f_{\mu\nu}$, $^{*}f_{\mu\nu}$ are orthogonal to n_{μ}. From this, it is clear that all terms in (1) with explicit derivatives vanish, since any n_{μ} must contract either with $f_{\mu\nu}$ or itself. Thus, only polynomials in F (and $^{*}F$) can remain. But as is well known, any scalar function in F can be written as one depending only on the invariants F^{2} and $F^{*}F$, both of which vanish here. (Alternately, the eigenvalues of $f_{\mu\nu}$ as a matrix are all null, therefore so is any polynomial in it.) The plane wave,

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‡ Gravitational plane waves do, however, describe a singularity-free, complete (but not asymptotically flat) space.

obeying Maxwell's equations, can be thought of as governed by the unperturbed Maxwell Lagrangian while the effective source $\langle J^\mu \rangle \equiv \delta \mathscr{L}^M_{\text{eff}}/\delta A_\mu$ due to vacuum polarization vanishes here.

The gravitational case is entirely analogous; a plane wave here is a solution of the source-free Einstein equations characterized by (Ehlers and Kundt 1962) a curvature (or Weyl) tensor depending on a null ($(\nabla u)^2 = 0$) coordinate u, so that a gradient of the curvature is equivalent to multiplication by a null vector. The expression corresponding to (1) is now, in terms of the curvature tensor $R_{\mu\nu\alpha\beta}$,

$$\mathscr{L}^E_{\text{eff}} \sim \sum R \ldots \nabla R \ldots \Box R \ldots R \ldots, \tag{2}$$

whatever the character of the underlying virtual pairs (including gravitons). Any explicit derivatives again vanish either when contracted together to form a null D'Alembertian or when contracted with a curvature index which leads to a form proportional to the curl of the Ricci tensor (by the Bianchi identities).

Non-derivative polynomials again vanish by the null-eigenvalue property of the plane wave's curvature tensor. A detailed proof of the vanishing of local plane wave invariants is given by Jordan *et al* (1960)†.

So far, we have neglected the problems of divergences in the coefficients of equations (1) and (2), and we have also omitted a possible cosmological term in (2). In the Maxwell case, the only divergent coefficient is that of the F^2 term, which contributes to amplitude renormalization only, at least for minimally coupled spinor and scalar charged particles. For gravitation, all coefficients in (2) will diverge, in general, but for the plane wave this is irrelevant. If one uses dimensional regularization as a cut-off method, then there will be no cosmological term when the virtual particles are massless. If, on the other hand, massive systems are included, $\sqrt{-g}$ contributions are in general unavoidable. These would no longer permit a plane wave amplitude. A possible way out is to demand that the renormalized cosmological constant vanishes by starting with an appropriate unrenormalized one for the virtual gravitons, or, if only quantized matter is considered, to have equally many fermion and boson fields so as to cancel their vacuum stress tensors‡. However, in the absence of a satisfactory renormalizable quantum gravity model, such delicate questions seem premature.

The plane wave is an example for which the linearized approximation and the full theory coincide in the form of their solutions (in appropriate gauges)§. This is also the case for a Yang–Mills plane wave where the *a priori* non-abelian solutions $F^a_{\mu\nu}(nx)$ are effectively reducible to the Maxwell form because all components point in the same isospin direction (T T Wu 1974, private communication). Thus, we can immediately use the Maxwell argument to see that there is no vacuum polarization for this model either, since for this wave all Yang–Mills invariants clearly reduce to Maxwell ones.

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‡ Supersymmetric systems provide a particularly interesting example (Zumino 1975).

§ Linearized form is of course not sufficient to avoid vacuum polarization: spherically symmetric solutions and constant fields are immediate counter examples.

Note added in proof. Gibbons (1975) has recently given an explicit calculation of vacuum polarization for a quantized scalar field. Using appropriate regularizations he concludes that the induced stress tensor vanishes for plane waves.

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